Measurement-assisted variational simulation of non-trivial quantum states

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Layout

- Motivation
- Measurement-based quantum computing (MBQC)
- Variational simulation of non-trivial quantum states
- Research question: Measurement-assisted QAOA as an efficient/better simulation?

Motivation

- Variational simulation of nontrivial quantum states with QAOA [HH19] requires $\mathcal{O}(L)$ circuit depth

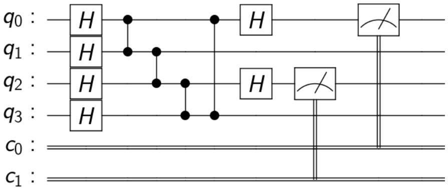
  Why? $\implies$ local unitaries spread correlations slowly, making nontrivial states expensive to prepare

- Entanglement + measurements can rapidly spread correlations (e.g. simulated the GHZ state with $\mathcal{O}(1)$ layer of measurements)

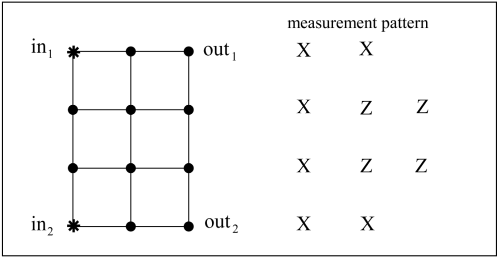
$\implies$ Entanglement + Measurements + Local unitaries = Speedup?

MBQC: One-way quantum computer [RB01]

Conventional quantum circuit models:

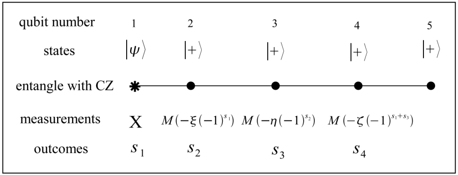


Cluster state: [Joz06] using quantum teleportation

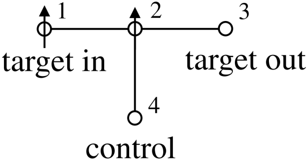


Universality: Quantum circuit model $\equiv$ Cluster state formulation.

- Transfer of information by teleportation
- Any single-qubit unitary can be done on a chain of qubits
- The CNOT gate can be implemented in a "T" configuration



(a) From [Joz06]



(b) From [RB01]

Variational simulation of non-trivial quantum states

QAOA [FGG14]: Quantum approximate optimization algorithm

- Principle: Quantum adiabatic theorem on $H = H_2 + H_1$
- Variational ansatz (modified in (2))

$$\boxed{|\psi(\boldsymbol{\gamma}, \boldsymbol{\beta})\rangle = \underbrace{e^{-i\gamma_p H_1} e^{-i\beta_p H_2} \dots e^{-i\gamma_2 H_1} e^{-i\beta_2 H_2} e^{-i\gamma_1 H_1} e^{-i\beta_1 H_2}}_{p \text{ layers}} |\psi_1\rangle} \tag{1}$$

  - $(\boldsymbol{\gamma}, \boldsymbol{\beta}) = (\gamma_p, \dots, \gamma_1, \beta_p, \dots, \beta_1)$
  - $|\psi_1\rangle$ = ground state of H_1 (easy to prepare)

- Cost function:

$$\text{Overlap: } |\langle\psi_0|\psi(\boldsymbol{\gamma}, \boldsymbol{\beta})\rangle|^2, \quad \text{or} \quad \text{Energy: } \langle\psi(\boldsymbol{\gamma}, \boldsymbol{\beta})| H |\psi(\boldsymbol{\gamma}, \boldsymbol{\beta})\rangle .$$

Variational simulation of the GHZ state

Example: GHZ state $\sim |0\rangle^{\otimes L} + |1\rangle^{\otimes L}$

$$H_{GHZ} = -\sum_{i=1}^{L} Z_i Z_{i+1} = \underbrace{-\sum_{i=1}^{L} Z_i Z_{i+1}}_{H_2} \underbrace{-0\sum_{i=1}^{L} X_i}_{H_1}, \qquad |GS_{H_1}\rangle = \bigotimes_{i=1}^{L} |+\rangle$$

$\implies$ Perfect fidelity, $p \sim L/2$.

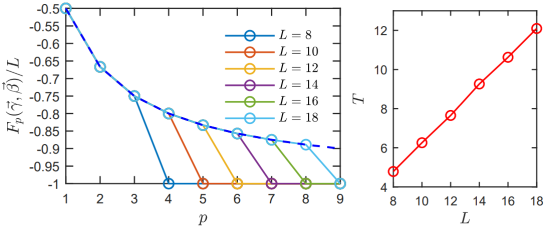


Figure: GHZ state simulation. Fidelity & p vs. L, [HH19]

Variational simulation of TFIM ground state

Example: Transverse field Ising model

$$H := H_2 + H_1 = -\sum_{i=1}^{L} Z_i Z_{i+1} - g \sum_{i=i}^{L} X_i$$

$\implies$ Perfect fidelity, $p \sim L/2$

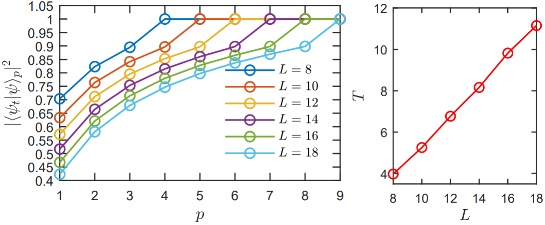


Figure: TFIM state simulation. Fidelity & p vs. L, [HH19]

Variational simulation...

Limitations of protocol in [HH19]:

- $p \sim L$.
- MERA construction [Vid08]: $p \sim \log(L)$,
  but non-local unitaries required.

$\Longrightarrow$ Is a measurement-assisted QAOA scheme a solution?

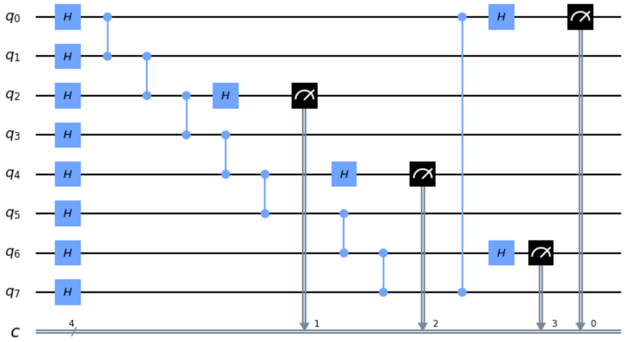


Figure: Preparing a 4-qubit GHZ state with a 8-qubit cluster state

Resulting state: $\sim |0\rangle^{\otimes 4} + |1\rangle^{\otimes 4}$ (up to one layer of Pauli corrections.)

Measurement-based QAOA for TFIM

Hamiltonian

$$H := H_2 + H_1 = -\sum_{i=1}^{L} Z_i Z_{i+1} - g \sum_{i=i}^{L} X_i$$

QAOA ansatz:

$$|\psi(\boldsymbol{\gamma}, \boldsymbol{\beta})\rangle = e^{-i\gamma_p H_1} e^{-i\beta_p H_2} \dots e^{-i\gamma_2 H_1} e^{-i\beta_2 H_2} e^{-i\gamma_1 H_1} e^{-i\beta_1 H_2} |\psi_1\rangle$$

MBQC is universal $\implies$ Measurement-based QAOA ansatz is possible.

Ingredients: Z, X-rotations, & $CNOT$.

♣ Scheme can be simplified by changing measurement pattern.

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But...

Limitations:

- $p \sim L$, where p is the number of layers of measurements.
- Non-local unitaries required

$\downarrow\downarrow\downarrow$

Two possibilities:

- QAOA is insufficient; need a completely new algorithm.
- QAOA is sufficient, but need better MBQC implementation. $\Longleftarrow$

2nd possibility: How far can QAOA go?

Test QAOA with TFIM without translation invariance:

$$\mathcal{H} = \sum_j J_j Z_j Z_{j+1} + \sum_j g_j X_j \tag{2}$$

Modified QAOA ansatz (reference (1))

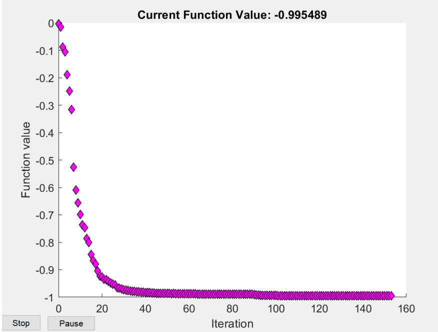
- p layers
- Each layer is parameterized by $(\boldsymbol{\gamma}, \boldsymbol{\beta})_k = (\gamma_1, \ldots, \gamma_L, \beta_1, \ldots, \beta_L)_k$.

Conjecture

This modified QAOA can target any point in the phase diagram with perfect fidelity for at most $p = L/2$. In which case, the total number of parameters is L^2.

2nd possibility: How far can QAOA go?

Conjecture seems to hold:



(a) $L = 8$, $p = 4$

```
Ground state energy
  -10.9885

First Optimal angles
    0.3740    0.6927    0.0761    0.7
    0.4267    0.9027    0.1085    0.5
    0.7873    0.2077    0.0500    0.8
    0.6822    0.2533    0.9213    0.5
    0.0766    1.2427    0.9784    0.6
    0.6315    0.1058    0.1814    0.0
    0.0972    0.2271    1.1599    0.4
    1.4486    1.1941    0.5562    0.5

First Fidelity by Overlap: 99.5489%
Time taken : 00:01:40
Time in sec: 100.2934
```

(b) 99.5% fidelity

Note: Fidelity here is limited by precision setting.

2nd possibility: How far can QAOA go?

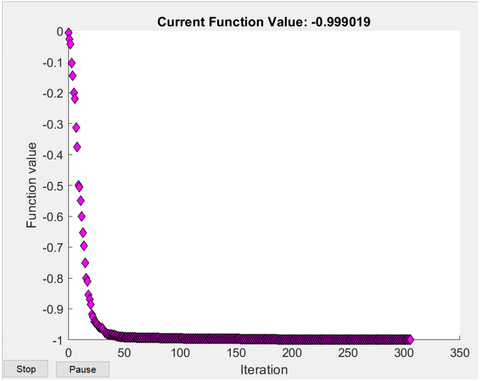
```
Ground state energy
  -17.0287

First Optimal angles
    0.8137    0.5871    0.8301    0.7233    0.3972    1.4465    0.8842    0.0169    0.6192    0.4545
    0.3606    0.0573    0.8608    0.6207    0.6431    0.5443    1.0091    0.9094    0.3803    0.7282
    1.0406    0.4527    0.1617    0.2719    0.4206    0.9273    0.1018    0.2185    0.3168    0.4171
    0.8567    0.6425    0.7548    0.7010    1.0682    1.3780    0.3439    1.2522    0.2124    0.1441
    1.0175    0.5888    0.2326    0.6124    0.7090    0.5247    0.6871    0.9656    0.2851    0.3794
    0.5176    0.6883    1.1344    0.2031    0.2970    1.0806    0.9063    0.0943    1.3235    0.2157
    0.8787    0.2169    0.7295    0.4612    0.3440    0.1942    0.7990    1.0653    0.0877    0.1932
    0.6477    0.7846    0.3086    0.1854    0.4175    0.3125    0.4870    0.7532    0.1419    0.7952
    0.9112    0.6097    0.5502    0.6992    0.1392    0.1485    0.0476    0.2726    0.0506    1.3016
    0.0247    0.4284    0.4669    0.3192    0.7880    0.5810    0.1528    0.1018    0.7222    0.0436

First Fidelity by Overlap: 99.9391%
Time taken : 00:09:14
Time in sec: 553.7107
```

Figure: $L = 10$, $p = 5$, 99.9% fidelity.

\# Parameters $\sim L^2 \implies$ need more computing power to test $L > 16$.

♣ Can simulate excited states with the same symmetry.

Example: 6th excited state for $L = 8$



(a) $L = 8$, $p = 4$, 6th excited state

```
Energy of state: 6
   -9.2687

First Optimal angles
    1.1817    0.2482    0.7576    0.
    0.2730    1.1437    1.1979    0.
    1.2387    0.7244    0.2111    0.
    0.5980    0.5456    0.2091    0.
    0.9507    0.3858    0.0844    0.
    0.6518    0.4670    0.3860    0.
    0.5786    0.5014    1.1874    0.
    1.0055    0.9423    0.6903    0.

First Fidelity by Overlap: 99.9019%
Time taken : 00:03:19
Time in sec: 199.2821
```

(b) 99.9% fidelity.

Measurement-assisted QAOA?

Consider the following algorithm:

- Make a random QAOA state from $\bigotimes |+\rangle$ on $2L$ qubits with:
    - Low-depth (sublinear?)
    - Randomly generated parameters
- Measure every other qubit $\implies$ get a L-qubit subsystem.
- Apply the QAOA optimization on this subsystem.

Intuition: Steps 1 and 2 generate a state with high entanglement, so that QAOA can drive it to the target state more quickly.

$\boxed{?}$: Quicker? How much quicker?

Summary & Questions

Summary

- MBQC & simulating the GHZ state
- QAOA & its "range"
- Possible measurement-assisted QAOA algorithm

Questions:

- Is it possible, in principle, to get speedup with MBQC + QAOA?
- Target the critical ground state ($g \equiv 1$) with sublinear circuit depth?

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- Timothy Hsieh, advisor (Perimeter)
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References I

- Edward Farhi, Jeffrey Goldstone, and Sam Gutmann, *A quantum approximate optimization algorithm*, arXiv preprint arXiv:1411.4028 (2014).
- Wen Wei Ho and Timothy H Hsieh, *Efficient variational simulation of non-trivial quantum states*, SciPost Phys **6** (2019), 029.
- Richard Jozsa, *An introduction to measurement based quantum computation*, NATO Science Series, III: Computer and Systems Sciences. Quantum Information Processing-From Theory to Experiment **199** (2006), 137–158.
- Michael A. Nielsen, *Cluster-state quantum computation*, Reports on Mathematical Physics **57** (2006), no. 1, 147 – 161.
- Robert Raussendorf and Hans J. Briegel, *A one-way quantum computer*, Phys. Rev. Lett. **86** (2001), 5188–5191.

References II

Guifré Vidal, *Class of quantum many-body states that can be efficiently simulated*, Physical review letters **101** (2008), no. 11, 110501.